

A278 Algebraic proofs

Q1.

Prove algebraically that the sum of the squares of any two consecutive even numbers is always a multiple of 4

(Total for question = 3 marks)

Q2.

Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

(Total for question is 4 marks)

Q3.

Prove algebraically that the difference between the squares of any two consecutive odd numbers is always a multiple of 8

(Total for question = 3 marks)

Q4.

Prove algebraically that the difference between the squares of any two consecutive integers is always an odd number.

(Total for question = 3 marks)

Q5.

For any three consecutive whole numbers, prove algebraically that the largest number and the smallest number are factors of the number that is one less than the square of the middle number.

(Total for question = 3 marks)

Q6.

Prove algebraically that

$(2n + 1)^2 - (2n + 1)$ is an even number

for all positive integer values of n .

(Total for question = 3 marks)

Q7.

n is an integer greater than 1

Prove algebraically that $n^2 - 2 - (n - 2)^2$ is always an even number.

(Total for question = 4 marks)

Q8.

Prove that the sum of the squares of any three consecutive odd numbers is always 11 more than a multiple of 12

(Total for question = 3 marks)

Q9.

Prove algebraically that the difference between any two different odd numbers is an even number.

(Total for question = 3 marks)

Q10.

Given that n can be any integer such that $n > 1$, prove that $n^2 - n$ is never an odd number.

(Total for question = 2 marks)

Q11.

The product of two consecutive positive integers is added to the larger of the two integers.

Prove that the result is always a square number.

(Total for question = 3 marks)

Q12.

n is an integer.

Prove algebraically that the sum of $\frac{1}{2}n(n+1)$ and $\frac{1}{2}(n+1)(n+2)$ is always a square number.

(Total for question = 2 marks)

Q13.

Here are the first five terms of an arithmetic sequence.

7 13 19 25 31

Prove that the difference between the squares of any two terms of the sequence is always a multiple of 24

(Total for question is 6 marks)

Q14.

a, b, c are positive integers such that $a > b > c$

N is the largest three digit number that has the digits a, b and c .

K is the smallest three digit number that has the digits a, b and c .

(a) Use algebra to show that the difference between N and K is always a multiple of 99

(3)

(b) If $a > b$ and $b = c$ will the difference between N and K still be a multiple of 99?

Justify your answer.

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(1)

(Total for question = 4 marks)